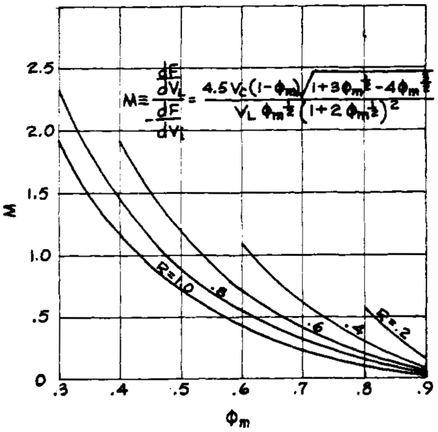

Fig. 2—A comparison of modulation sensitivities of the focus lens oscillator and a conventional two-cavity klystron oscillator. F is the frequency of oscillation, V_i is the cathode voltage of the conventional oscillator, and V_c is the cathode voltage of the focus lens oscillator. The curve parameter R is the ratio of lens to cathode voltage.

changes; consequently, this type oscillator gives an order of magnitude improvement over an ion focused two-cavity klystron oscillator in regard to thermal tuning effects. Also, by proper choice of feedback coupling between cavities and tuning of the cavities, it is possible to make this oscillator have a flat top-mode shape which means that the frequency modulation is accompanied by a minimum of residual amplitude modulation.

J. Robert Ashley
Sperry Electronic Tube Div.
Sperry Rand Corp.
Gainesville, Fla.

Comments on "Penetration of the Ionosphere by Very-Low-Frequency Radio Signals—Interim Results of the LOFTI I Experiment"*

In their paper Leiphart, *et al.*,[1] have stated that "V.L.F. radio waves will propagate with least absorption when the angle of inclination θ of the wave normal relative to the geomagnetic field is zero or 180°." Subsequently they calculate the "least possible loss" on this basis ($\theta=0$).

Now, if a satellite is situated at a given point in the ionosphere, the signal energy will reach it along the "ray path," which is determined by two conditions. First, Snell's law gives

$$n \sin \beta = \text{const.} \qquad (1)$$

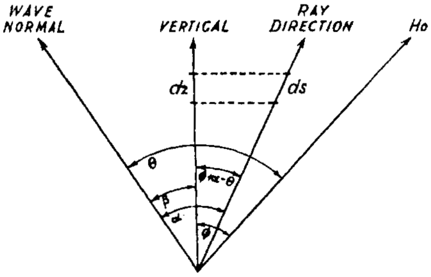

Fig. 1—Angular relations used in text.

(see Fig. 1 which defines the angles used here) where n, the complex refractive index, defined by the Appleton-Hartree equation, is a function of θ and contains the "real" refractive index μ, and the absorption index χ:

$$n = \mu(\theta) - i\chi(\theta). \qquad (2)$$

For typical daytime conditions μ may vary between 20 and 100 in the E and F layers, so that the wave normal will rapidly approach the vertical direction. Secondly, the angle α between the wave normal and the ray direction is given (Budden[2]) by

$$\tan \alpha = \frac{1}{2}\,\frac{X/Y_L}{1+X/Y_L}\tan\theta \qquad (3)$$

where X and Y_L have their usual meaning in magneto-ionic theory and depend on the ionospheric model chosen. The ray path being determined, we may now calculate the factor A by which the field vectors in the ray wave packet are attenuated[2]

$$A = \exp\left[\frac{\omega}{c}\int \chi_R ds\right] \qquad (4)$$

where ω is the wave angular frequency, c the speed of light, χ_R the ray absorption index, the integration being performed along the ray path s.

In general we have[2]

$$\chi_R = \chi(\theta) \cos \alpha \qquad (5)$$

and the attenuation becomes

$$A = \exp\left[\frac{\omega}{c}\int \chi(\theta)\cos\alpha \cdot \sec(\phi+\alpha-\theta)dz\right] \qquad (6)$$

integrating vertically from the point of entry of the wave packet in the lower D region to the height of the satellite for points on the ray path.

Eq. (6) simplifies in two cases. First, if the ionization is field aligned rather than horizontally stratified, then $\theta=0$, $\alpha=0$, and (6) becomes

$$A = \exp\left[\frac{\omega}{c}\int \chi(0)ds\right]. \qquad (7)$$

Alternately, if the wave normal is vertical, ($\theta=\phi$), then

$$A = \exp\left[\frac{\omega}{c}\int \chi(\phi)dz\right]. \qquad (8)$$

The analysis performed by Leiphart, *et al.*, ($\theta=0$), is thus justifiable only for regions of field-aligned ionization. But this can hardly be considered to be a typical condition in the D layer where most of the absorption loss occurs.

The statement that the condition $\theta=0$ yields minimum absorption should likewise be examined. In general $\chi(\theta)$ is indeed a minimum for $\theta=0$, having a particularly simple form above some 90-km height,

$$\chi(\theta) \propto (\cos\theta)^{-3/2}. \qquad (9)$$

However, since the absorption of a wave packet between two fixed heights depends on the path length, the condition for minimum absorption will be that for which the integral in (6) is a minimum. For simplicity we will minimalize the integrand $[\chi(\theta)\cos\alpha \sec(\phi+\alpha-\theta)]$, and for this purpose two distinct regions are distinguished.

Above some 90-km height $X/Y_L \gg 1$ and (3) becomes

$$\tan\alpha = (\tan\theta)/2. \qquad (10)$$

Substituting (9) and (10) into (6) it is found that the integrand is a minimum for a value of θ for which the expression

$$\cos^{1/2}\theta[3\cos\phi + \cos(2\theta-\phi)]$$

is a maximum. This expression is independent of α and the maximum is determined by the angle ϕ in a given region. For a geomagnetic latitude of 35° we find minimum absorption between any two heights in the E and F layers for $\theta=12°$, the corresponding figure for latitude 20° being $\theta=20°$.

In the lower D region the minimum value of the integrand in (6) may be determined numerically. At 70-km height, for instance, for a solar zenith angle of 60°, and geomagnetic latitude 35°, we find a minimum at $\theta=23°$, using recent D-layer models and collision frequencies. Similar results are obtained throughout the D region with minimum absorption being obtained in all cases for $\theta>0$.

C. Altman
H. Cory
Physics Department
Technion, Israel Inst. of Tech.
Haifa, Israel

Author's Reply[3]

These remarks are in general agreement with results of the continuing analysis of this experiment. While we assumed in the preliminary analysis phase of LOFTI that the minimum absorption of VLF energy in the ionosphere will occur when the angle between the wave normal and the direction of the magnetic field is zero, later study has indicated, as Altman and Cory have demonstrated, that this assumption provides only a rough approximation of this element of the total propagation loss. A more thorough analysis has been under way for some time and a machine program is being prepared to compute absorption of VLF energy in the space between a terrestrial transmitter and a receiver in the ionosphere. We hope to publish this analysis upon its completion together with a comparison with measured LOFTI I data.

J. Plumer Leiphart
Naval Research Lab.
Washington, D. C.

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* Received June 18, 1962; revised manuscript received July 18, 1962.

[1] J. P. Leiphart, R. W. Zeek, L. S. Bearce, and E. Toth, "Penetration of the ionosphere by V.L.F. radio signals—interim results of the LOFTI I experiment," Proc. IRE, vol. 50, pp. 6–17; January, 1962.

[2] K. G. Budden, "Radio Waves in the Ionosphere," Cambridge University Press, London, England; 1961.

[3] Received October 2, 1962.